Ru Ye Jul 29, 2020

Permissivism and the Accuracy-Conduciveness of Rationality

1. Introduction

Permissivism: Sometimes there are multiple rational responses to the same body of evidence. (In Bayesian terms: there are multiple rational priors probabilities.)
Impermissivism: There is always only one rational response to a body of evidence. (In Bayesian terms: there is a unique rational priors probability.)

Value Problem for Permissivism:

(1) We think rationality is valuable because it's accuracy-conducive, and yet
(2) We can only make sense of the accuracy-conduciveness of rationality if we assume rationality is impermissive.

My Plan:

Solve the problem by arguing that it rests on a false understanding of the rationality-accuracy connection; There is a more attractive understanding of the connection and it's compatible with Permissivism.

2. The value problem.

2.1 *An account of the accuracy-conduciveness of rationality.*

Rational credence *tend* to be more accurate than irrational credence;
The tendency could be cashed out in terms of what happens *on average*;
This 'on average' could be expressed as what happens in expectation relative to some probability function. More precisely:

(W, F, Pr): W is a set of possible worlds; F is an algebra on W, and Pr is a set of probability functions defined on F.
$P \in Pr$: a Bayesian agent.
A: Pr $\times$ W$\rightarrow$ R: the agent's accuracy function.
$EA_p(Cr) = \sum_{w \in W} p(w)A(Cr, w)$: the agent P's expectation of Cr's accuracy score.

$E \subset W$: a body of evidence of some agent.
Cr_E: a rational credence given E;
$Cr^*_{E:}$ an irrational credence given E.

'expectation understanding' of the rationality-accuracy connection: $EA_p(Cr_E) > EA_p(Cr^*_E)$
Question: what's the probability function *p*?
Answer: a rational agent with evidence E.

Expectation Constraint

For any rational person C whose total evidence is E and who is certain that credence Cr is rational given E and Cr* is irrational given E, the person regards Cr more expectedly accurate than Cr*. That is, $EA_c(Cr_E) > EA_c(Cr^*_E)$.

2.2 Permissivism is incompatible with Expectation Constraint.

If Permissivism were true, then consider:

Range of rational credence in H given your evidence = [0.6, 0.9].
You: 0.9 (a rational credence)
A rational credence given your evidence: 0.6
An irrational credence: 0.95.
You regard 0.6 *less* expectedly accurate than 0.95!

So, from a rational point of view (namely, your point of view), a rational credence (0.6) is *less* expectedly accurate than an irrational credence (0.95).

3. Convergence-to-Truth as Accuracy-Conduciveness

3.1 Motivating a Convergence Approach to Accuracy-Conduciveness

A less expectedly accurate credence in your evidential situation might perform better (than a more accurate credence) if given more and more evidence.

Example:
Rational range for H: [0.1, 0.9].
You: 0.1
A rational credence: 0.9
An irrational credence: 0
Which is more expectedly accurate? 0
But which do you think will perform better if given more and more evidence?
The 0 person will *not* converge to truth in a H world; the 0.9 person could.

Convergence Constraint

A rational credence function performs better than an irrational credence function in convergence to truth.

3.2 Clarifying Convergence Constraint

An agent who tosses a coin for infinitely many times: (W, F, P).
w ∈ W: a countably infinite binary sequence representing a possible outcome of the infinite coin toss such as (HHTH….),
F: a σ-algebra on W, representing the set of propositions the agent considers during the learning.
P: is a probability function from F to [0, 1], representing the agent's priors.
$E_{nw} \subset W$: the agent's evidence in w after n tosses.
$I_A(w)$: Indicator function for A in F.
The agent converges to the truth in world w as he gets more evidence: for any A in F,

Convergence to truth in a world: $\lim_{n\to\infty} P(A \mid E_{nw}) = I_A(w)$

Modes of Convergence:

Everywhere: converges to truth in all worlds in W;
Almost surely: guaranteed to converge to truth everywhere from the point of view of some probability function P* that is also defined on F.
Speed of convergence: a convergence can happen fast or slow.
Domain of convergence: a convergence can happen everywhere or 'almost everywhere'
Uniformity of convergence: a convergence can be uniform or merely point-wise

Convergence Constraint*

A rational credence function performs better than an irrational credence function in convergence to truth, in the sense that it achieves all the desirable modes of convergence that can be achieved whereas an irrational credence doesn't.

*3.3 Defending Convergence Constraint**

(1) Intuitive:
A credence function encodes a learning method;
A natural way of testing whether a learning method is good: Look at how well the method performs in a highly idealized setting where one can gain infinite amount of evidence.
If you fail to get the truth even on this highly idealized setting, then there is something intrinsically wrong with your learning method.

(2) Powerful:
Can be used to vindicate our familiar norms of rationality, such as regularity (Lewis), simplicity (Lin), enumerative induction (Lin), and Reflection (Nielsen).

In contrast, Expectation Constraint is not able to vindicate any norm of rationality. (Expected-accuracy arguments for norms of rationality don't rely on Expectation Constraint.)

3.4 Convergence Constraint Is Compatible With Permissivism.

(1) Distance is not a problem: From a rational point of view, a closer credence doesn't necessarily have better performance in converging to truth.
(2) So, if Convergence Constraint is incompatible with Permissivism, it cannot be due to a reason that is similar to the reason why Expectation Constraint is incompatible with Permissivism.
(3) Are there any other reasons to think that they are incompatible? Here is a general reason for a negative answer:
Accuracy score of a credence function in a world is extremely sensitive to each particular value the function assigns to propositions, such that if you change your credence on a proposition even a little your accuracy score would change. But performance in convergence is not so sensitive: for an infinite sequence, a change at some point won't affect its general trend. Therefore, two different credence functions might have the same asymptotic behavior.

4. To conclude:

(1) The value of rationality in pursuing accuracy doesn't lie in that rational credence is more expectedly accurate than irrational credence from some rational point of view, but that rational credence performs better in convergence to truth;

(2) This convergence understanding of the rationality-accuracy connection is promising and it is compatible with Permissivism.

So, Permissivists can happily say that rationality is indeed accuracy-conducive.